2018 Enrolment The 2nd

Japan University Examination

Advanced Mathematics

Examination Date: May 2017

(90 min)

Do not open the examination booklet until the starting signal for the exam is given. Please read the following instructions carefully.
Please fill in the examinee no. and name below.

Instructions

1. The booklet contains 3 pages.
2. The answer sheet is one piece of one sided paper.
3. In the case that you notice there are parts in the booklet where the print is not clear or there are missing pages or misplaced pages, or the answer sheet is soiled, raise your hand to report to the invigilator.
4. There are 3 questions to be answered.
5. Fill the examinee no. and name in the answer sheet.
6. Use black pencil to write answers in the designated section in the answer sheet.
7. Memos and calculations can be written on the examination booklet.
8. When the signal to end the exam is given, check again to see that the examinee no. and name is filled in and submit the answer sheet and the examination booklet according to the invigilator's instructions.

Examinee's No.	Name

1 It is known that the arithmetic progression $\{a_n\}$ satisfy the following (A) and (B):

(A) $a_6 = 12$
(B) $a_{15} = 5a_3$

(1) Write down the first item and the common difference of $\{a_n\}$, and use n to express the n^{th} item a_n.

(2) Calculate the sum from the first item to the tenth item of $\{a_n\}$.

(3) It is known that in the group sequence, there is 1 2^{a_1} in the first group, and 2 2^{a_2} in the second group,3 2^{a_3} in the third group… then m (m=1, 2, 3, …) 2^{a_m} in the m^{th} group as following,

1st group	2nd group	3rd group	4th group	…
2^{a_1}	2^{a_2}, 2^{a_2}	2^{a_3}, 2^{a_3}, 2^{a_3}	2^{a_4}, 2^{a_4}, 2^{a_4}, 2^{a_4}	2^{a_5}, …

Set that the last item of the m^{th} group is b_m,then the last item of the 1^{st} group is b_1, the last item of the 2^{nd} group is b_2, the last item of the 3^{rd} group is b_3……and so on.

(i) Express the m^{th} of the arithmetic progression b_m with m.
(ii) Calculate the sum of $b+b+b$ ・・・+ b_m with m.
(iii) If count from 2^{a_1}, b_m will be ranked as?
(iv) Calculate the sum from 2^{a_1} to b_m.

2 Here are 2 circles on plane . The expressions are as following:

$$C_1: x^2 + y^2 - 2x - 6y + 9 = 0$$
$$C_2: x^2 + y^2 - 8ax - 6ay + 21a^2 = 0 \quad (a \text{ is a positive constant})$$

The center of C_1 is A, and the center of C_2 is B.

(1) Calculate the length of line segment AB with a.

(2) Assuming that C_1 and C_2 are circumscribe d with each other, calculate all the possible value of a.

(3) Setting the maximum value in (2) for a:

(i) Calculate the coordinate of the tangent point of C_1 and C_2.

(ii) Calculate the expression of the straight lines which tangent to both C_1 and C_2 at the same time.

(iii) Calculate the area which surrounded by the straight lines from (ii).

3 It is known that the origin of plane xy is O, and there is an ellipse C_1 and a hyperbola C_2 on plane xy. The expressions are as following:

$$\text{ellipse } C_1:\ \frac{x^2}{8}+\frac{y^2}{2}=1,$$

$$\text{hyperbola } C_2:\ \frac{x^2}{4}-y^2=k \quad (k \text{ is a positive constant})$$

(1) Calculate the long axes and short axes of C_1 and the coordinates of focal point.

(2) Assuming that the intersection points of the asymptote of C_2 and C_1, the point in the first quadrant is A. Calculate the coordinates of point A, and write down the expression of the tangent line l of C_1 which goes through point A.

(3) Assuming that C_2 goes through the intersection point of l and x-axis which described in (2), then

(i) Calculate the value of k.

(ii) Assuming that point P is on C_2, OP= p. The focal points of C_2 are F, F′. Calculate the product of the length of two lines PF・PF′.